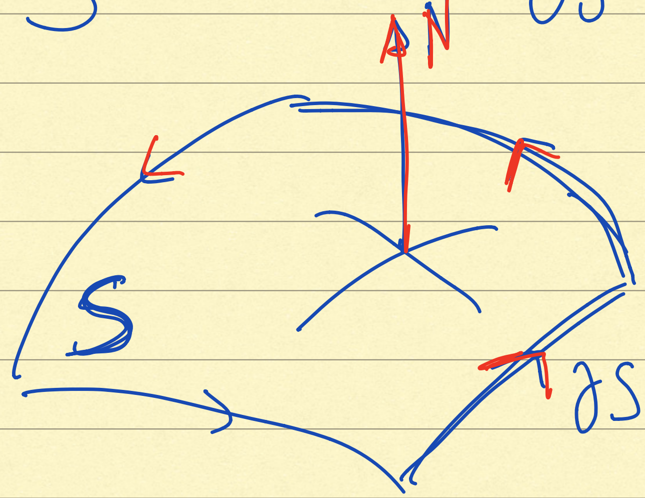
Teorema de Stokes.

$$\underbrace{\iint_S \operatorname{rot} F \cdot N}_{\text{Fluxo}} = \underbrace{\oint_{\partial S} F \cdot dg}_{\text{Trabalho}}$$

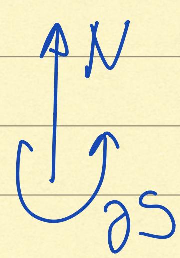


$\partial S \equiv$ bordo de S

$F : \mathbb{R}^3 \to \mathbb{R}^3$, C^1

$S \subset \mathbb{R}^3$ superfície $\boxed{\text{orientável}}$

(regra da mão direita):



$$\operatorname{rot} F = \text{"}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}\text{"} \begin{array}{l} \\ \leftarrow \nabla \\ \leftarrow F \end{array}$$

$$\operatorname{rot} F \equiv \boxed{\nabla \times F.} \checkmark$$

Exercício 1 (F. 13)

$$\iint_S \operatorname{rot} F \cdot N = ???$$

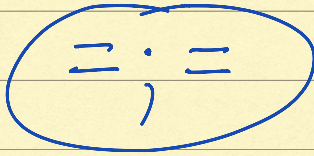
$$\boxed{N_z < 0}$$

$$S: \ 0 < z = x^2 + y^2 - 1 < 3$$

$$F(x,y,z) = \left(y, -x, \cos(x^2+z^2)\right)$$

Solução: Calcular $\oint_{\partial S} F \cdot dg$

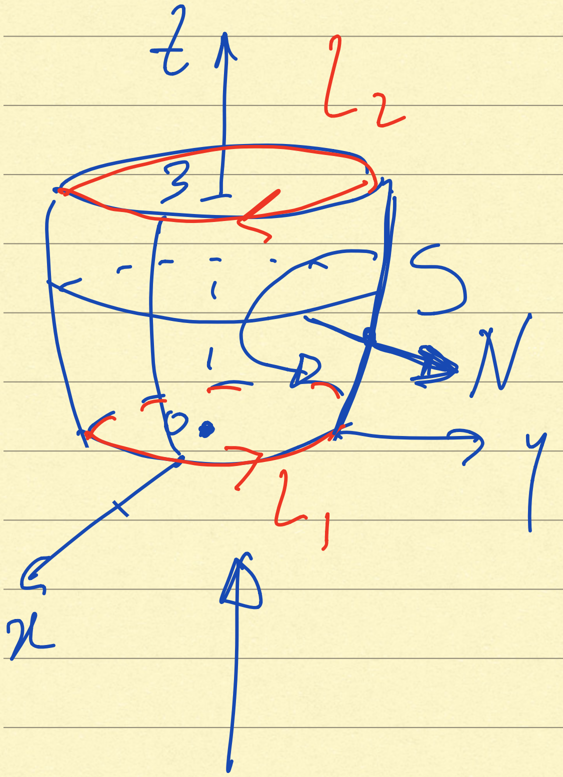
$S \longrightarrow \partial S$

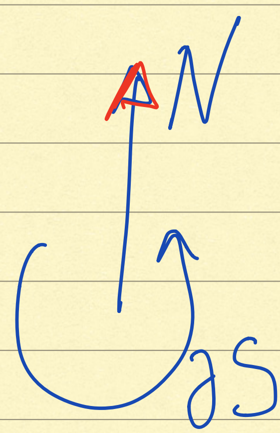
$$\begin{cases} z = x^2+y^2-1 \\ z > 0 \\ z < 3 \end{cases}$$

$L_1 : \begin{cases} z = x^2+y^2-1 \\ z = 0 \end{cases}$

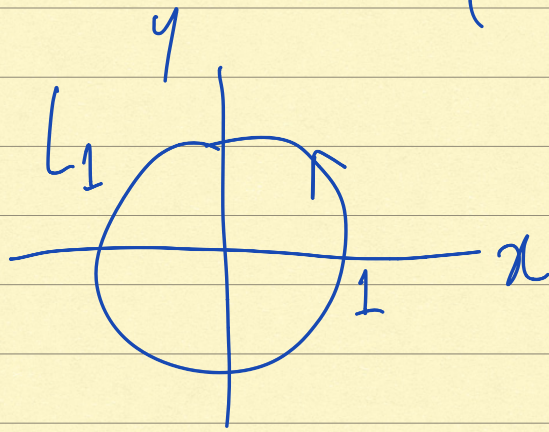
$L_2 : \begin{cases} z = x^2+y^2-1 \\ z = 3 \end{cases}$



$N_z < 0$



$$L_1 : \begin{cases} z = x^2 + y^2 - 1 \\ z = 0 \end{cases} \qquad \begin{cases} x^2 + y^2 = 1 \\ z = 0 \end{cases}$$



$$g(t) = (\cos t, \operatorname{sen} t, 0) \qquad 0 \le t \le 2\pi$$

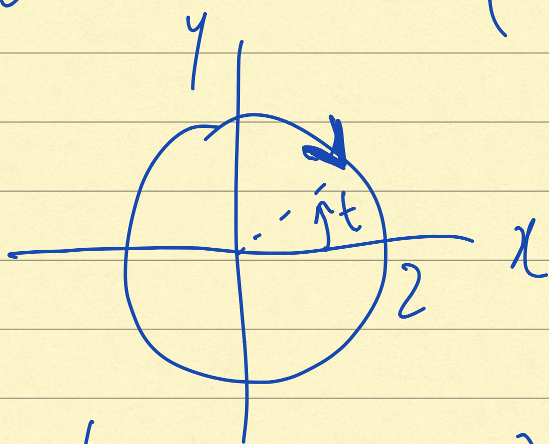
$$g'(t) = (-\operatorname{sen} t, \cos t, 0)$$

$$F(g(t)) = (\operatorname{sen} t, -\cos t, \ldots)$$

$$F(g(t)) \cdot g'(t) = -\operatorname{sen}^2 t - \cos^2 t = -1$$

$$\int_{L_1} F \cdot dg = \int_0^{2\pi} (-1)\, dt = -2\pi \,//$$

$$L_2 : \begin{cases} z = x^2+y^2-1 \\ z = 3 \end{cases} \quad \begin{cases} x^2+y^2 = 4 \\ z = 3 \end{cases}$$



$$g(t) = (2\cos t, -2\operatorname{sen} t, 3) \qquad 0 \leq t \leq 2\pi$$

$$g'(t) = (-2\operatorname{sen} t, -2\cos t, 0)$$

$$F(g(t)) = (-2\operatorname{sen} t, -2\cos t, \ldots)$$

$$F(g(t)) \cdot g'(t) = 4\operatorname{sen}^2 t + 4\cos^2 t = 4$$

$$\int_{L_2} F \cdot dg = \int_0^{2\pi} 4\,dt = 8\pi$$

$$\iint_S \operatorname{rot} F \cdot N \overset{\text{stokes}}{=} \oint_{\partial S} F \cdot d\gamma =$$

$$= \int_{L_1} F \cdot d\gamma + \int_{L_2} F \cdot d\gamma = -2\pi + 8\pi$$

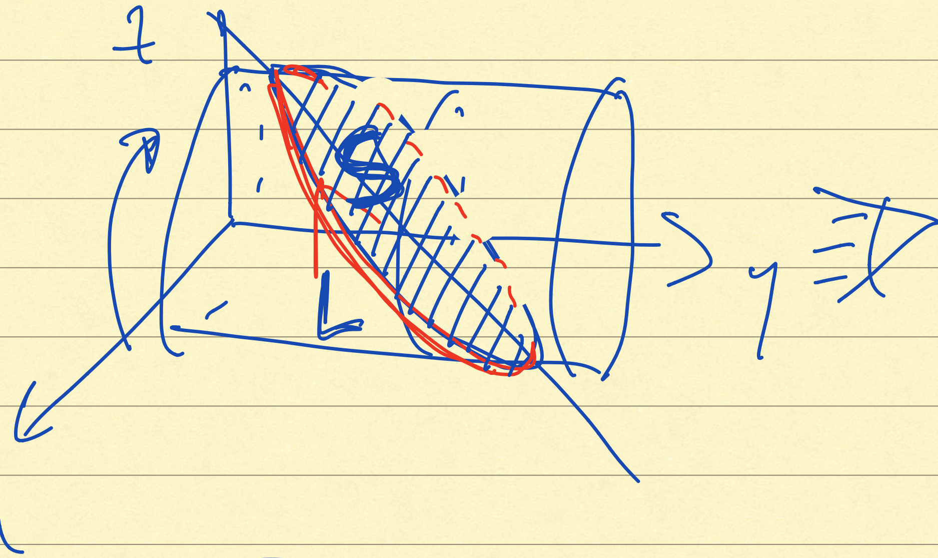
$$= 6\pi$$

Exercício 2 (F. 13)

$$\int_L G \cdot d\gamma = ?$$

$$G(x,y,z) = (x, -z, y + z^2)$$

$$L : x^2 + z^2 = 1 \; ; \; y + z = 1$$



$L: \; x^2 + z^2 = 1 \; ; \; \boxed{y + z = 1}$

<u>Stokes</u>: $\displaystyle\int_{L} G \cdot dg = \iint_{\boxed{S}\,?} \mathrm{rot}\, G \cdot N$

$L \equiv \partial S$

de L construir S tal que

$L = \partial S$

$L \longrightarrow S$ limitada

$=$

$=$

$=$

$\left\{\begin{array}{l} x^2+z^2=1 \\ y+z=1 \end{array}\right. \longleftrightarrow \boxed{\begin{array}{l} x^2+z^2<1 \\ \boxed{y+z=1} \end{array}}$

$$\boxed{L = \partial S}$$